Extension of Collins hard limiter paper to case where the noise characteristics of each hard limiter stage are not identical.

In practice the noise characteristics of each stage of a cascade of filtered limiter amplifiers may not be identical.

Starting with Equations 19 and 20 from the Collins paper:

$$J^2 = \frac{N_0}{4V\rho_{\text{in}}} QM \qquad (19)$$

where M, the normalized square jitter, is given by

$$\begin{aligned} M &= g_1 + \frac{g_1 g_2}{g_1^2} + \frac{g_1 g_2 g_3}{g_1^2 g_2^2} + \cdots + \frac{g_1 g_2 \cdots g_n}{g_1^2 g_2^2 \cdots g_{n-1}^2} \\ &= g_1 + \frac{g_2}{g_1} + \frac{g_3}{g_1 g_2} + \cdots + \frac{g_n}{g_1 g_2 \cdots g_{n-1}}. \end{aligned} \qquad (20)$$

with the auxiliary minimisation problem: minimise

$$M' = g_{n-1} + \frac{g_n}{g_{n-1}} \quad \text{subject to} \quad g_{n-1} g_n = \text{constant}. \qquad (21)$$

The solution to this minimization problem is

$$g_{n-1} = \sqrt{2g_n}. \qquad (22)$$

MODIFICATIONS

When Q, V, N_0 differ for each stage

$$\mathbf{J^2 = \sum(N_{0j}Q_jg_j)/(4V_j\rho_j)} \qquad \mathbf{(23)}$$

where

$\mathbf{N_{0j}}$	is the square of the equivalent input voltage noise spectral density (N_o) for limiter j
$\mathbf{Q_j}$	is Q for limiter j
$\mathbf{g_j}$	is the slope gain of limiter stage j
$\mathbf{V_j}$	is the pp clamp voltage for limiter stage j
$\boldsymbol{\rho_j}$	is the input slew rate for limiter stage j

Equation 23 can be re written as

$$\mathbf{J^2 = ((N_{01}Q_1)/(4V_1\rho_{in}))M} \qquad \mathbf{(24)}$$

where

$$\mathbf{M = g_1 + (g_2/ g_1)(N_{02}/N_{01})(Q_2/Q_1)(V_1/V_2) + (g_3/ g_1g_2)(N_{03}/N_{01})(Q_3/Q_1)(V_1/V_3) + ...}$$

or

$$\mathbf{M = g_1 + (g_2/ g_1)\varepsilon_1 + \varepsilon_2(g_3/ g_1g_2) + ... + \varepsilon_n g_n/(g_1g_2 ... g_{n-1})} \quad \textbf{(25)}$$

where

$$\boldsymbol{\varepsilon_j} \mathbf{= (N_{0j}/N_{01})(Q_j/Q_1)(V_1/V_j)} \quad \textbf{(26)}$$

with the auxiliary minimisation problem: minimise

$$\mathbf{g_{n-1} + \boldsymbol{\gamma}_n(g_n/g_{n-1})} \quad \textbf{(27)}$$

subject to the constraint

$$\boldsymbol{g_n g_{n-1} = constant}$$

Where

$$\boldsymbol{\gamma}_n \mathbf{= (N_{0n}/N_{0n-1})(Q_n/Q_{n-1})(V_{n-1}/V_n)} \quad \textbf{(28)}$$

or

$$\boldsymbol{\gamma}_n \mathbf{= (\boldsymbol{\varepsilon}_n /\boldsymbol{\varepsilon}_{n-1})} \quad \textbf{(29)}$$

The solution to which is

$$\mathbf{g_{n-1}^2 = 2\boldsymbol{\gamma}_n g_n} \quad \textbf{(30)}$$

or

$$\boldsymbol{g_{n-1} = \sqrt{(2\gamma_n g_n)}}$$

by induction:

$$\boldsymbol{g_{i-1} = \sqrt{(2\gamma_i g_i)} \qquad i = 1, 2, 3, ... , n.}$$

Slope gain calculation for multistage limiters

2 Stage limiter

$$g_1g_2 = g$$

$$g_1^2 = 2\gamma_2 g_2$$

$$g_1^3 = 2\gamma_2 g$$

3 stage limiter

$$g_1g_2g_3 = g$$

$$g_2^2 = 2\gamma_3 g_3$$

$$g_1^2 = 2\gamma_2 g_2$$

$$g_1^7 = 2^4\gamma_2^3\gamma_3 g$$

4 stage limiter

$$g_1g_2g_3g_4 = g$$

$$g_3^2 = 2\gamma_4 g_4$$

$$g_2^2 = 2\gamma_3 g_3$$

$$g_1^2 = 2\gamma_2 g_2$$

$$g_1^{15} = 2^{11}\gamma_2^7\gamma_3^3\gamma_4 g$$

5 stage limiter

$$g_1g_2g_3g_4g_5 = g$$

$$g_4^2 = 2\gamma_5 g_5$$

$$g_3^2 = 2\gamma_4 g_4$$

$$g_2^2 = 2\gamma_3 g_3$$

$$g_1^2 = 2\gamma_2 g_2$$

$$g_1^{31} = 2^{26}\gamma_2^{15}\gamma_3^{7}\gamma_4^{3}\gamma_5 g$$

6 stage limiter

$$g_1g_2g_3g_4g_5g_6 = g$$

$$g_5^2 = 2\gamma_6 g_6$$

$$g_4^2 = 2\gamma_5 g_5$$

$$g_3^2 = 2\gamma_4 g_4$$

$$g_2^2 = 2\gamma_3 g_3$$

$$g_1^2 = 2\gamma_2 g_2$$

$$g_1^{63} = 2^{57}\gamma_2^{31}\gamma_3^{15}\gamma_4^{7}\gamma_5^{3}\gamma_6 g$$

The calculation of the filter time constants and voltage gain of each stage from its slope gain is the same as in the Collins paper. Only the calculation of the slope gain and the jitter differ.

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