Extension of Collins hard limiter paper to case where the noise characteristics of each hard limiter stage are not identical.

In practice the noise characteristics of each stage of a cascade of filtered limiter amplifiers may not be identical.

Starting with Equations 19 and 20 from the Collins paper:

$$J^2 = \frac{N_0}{4V\rho_{\text{in}}} QM \qquad (19)$$

where M, the normalized square jitter, is given by

$$\begin{aligned} M &= g_1 + \frac{g_1 g_2}{g_1^2} + \frac{g_1 g_2 g_3}{g_1^2 g_2^2} + \cdots + \frac{g_1 g_2 \cdots g_n}{g_1^2 g_2^2 \cdots g_{n-1}^2} \\ &= g_1 + \frac{g_2}{g_1} + \frac{g_3}{g_1 g_2} + \cdots + \frac{g_n}{g_1 g_2 \cdots g_{n-1}}. \end{aligned} \qquad (20)$$

with the auxiliary minimisation problem: minimise

$$M' = g_{n-1} + \frac{g_n}{g_{n-1}} \quad \text{subject to} \quad g_{n-1}g_n = \text{constant}. \qquad (21)$$

The solution to this minimization problem is

$$g_{n-1} = \sqrt{2g_n}. \qquad (22)$$

MODIFICATIONS

Equation 20 is modified when the noise of each stage is not the same:

$$M = g_1 * k_1 + (g_2/g_1) * k_2 + (g_3/g_1 g_2) * k_3 + \ldots..$$

where

$k_1 = 1$
$k_2 = N_2/N_1$
$k_3 = N_3/N_1$

$N_1 = N_0$ is the equivalent input voltage noise power spectral density of the first stage

N_2 is the equivalent input voltage noise power spectral density of the second stage

N_3 is the equivalent input noise power spectral density of the third stage

The auxiliary minimisation problem becomes: minimise

$$M' = g_{n-1} + (g_n/g_{n-1})*\varepsilon_n$$

where

$$\varepsilon_n = N_n/N_{n-1}$$

subject to the constraint:

$$g_n g_{n-1} = constant$$

the solution is:

$$g_{n-1} = (2\varepsilon_n g_n)^{0.5}$$

by induction:

$$g_{i-1} = (2\varepsilon_i g_i)^{0.5} \qquad i = 1, 2, 3, ... , n.$$

Slope gain calculation for multistage limiters

2 Stage limiter

$$g_1 g_2 = g$$

$$g_1^2 = 2\varepsilon_2 g_2$$

$$g_1^3 = 2\varepsilon_2 g$$

3 stage limiter

$$g_1 g_2 g_3 = g$$

$$g_2^2 = 2\varepsilon_3 g_3$$

$$g_1^2 = 2\varepsilon_2 g_2$$

$$g_1^7 = 2^4 \varepsilon_2^3 \varepsilon_3 g$$

4 stage limiter

$$g_1 g_2 g_3 g_4 = g$$

$$g_3^2 = 2\varepsilon_4 g_4$$

$$g_2^2 = 2\varepsilon_3 g_3$$

$$g_1^2 = 2\varepsilon_2 g_2$$

$$g_1^{15} = 2^{11}\varepsilon_2^{7}\varepsilon_3^{3\varepsilon}{}_4 g$$

5 stage limiter

$$g_1 g_2 g_3 g_4 g_5 = g$$

$$g_4^2 = 2\varepsilon_5 g_5$$

$$g_3^2 = 2\varepsilon_4 g_4$$

$$g_2^2 = 2\varepsilon_3 g_3$$

$$g_1^2 = 2\varepsilon_2 g_2$$

$$g_1^{31} = 2^{26}\varepsilon_2^{15}\varepsilon_3^{7}\varepsilon_4^{3}\varepsilon_5 g$$

6 stage limiter

$$g_1 g_2 g_3 g_4 g_5 g_6 = g$$

$$g_5^2 = 2\varepsilon_6 g_6$$

$$g_4^2 = 2\varepsilon_5 g_5$$

$$g_3^2 = 2\varepsilon_4 g_4$$

$$g_2^2 = 2\varepsilon_3 g_3$$

$$g_1^2 = 2\varepsilon_2 g_2$$

$$g_1^{63} = 2^{57}\varepsilon_2^{31}\varepsilon_3^{15}\varepsilon_4^{7}\varepsilon_5^{3}\varepsilon_6 g$$

Case II: Where the k for each stage and consequently the Q's differ the problem is to

The auxiliary minimisation problem becomes: minimise

$$M'' = Q_{n-1}g_{n-1} + (g_n/g_{n-1})*(Q_nN_n)/(N_{n-1}\varepsilon_n)$$

or

$$M'' = g_{n-1} + (g_n/g_{n-1})*\varepsilon_n$$

where

$$\varepsilon_n = (N_n/N_{n-1})*(Q_n/Q_{n-1})$$

subject to the constraint:

$$g_ng_{n-1} = constant$$

the solution is:

$$g_{n-1} = (2\varepsilon_ng_n)^{0.5}$$

by induction:

$$g_{i-1} = (2\varepsilon_ig_i)^{0.5} \qquad i = 1, 2, 3, ... , n.$$

The calculation of the filter time constants and voltage gain of each stage from its slope gain is the same as in the Collins paper. Only the calculation of the slope gain and the jitter differ.

Bruce Griffiths